Numerical solution of Hamilton Jacobi Bellmann (HJB) equations by Tree Based Tensor Networks (HT/TT)

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Abstract

In this talk we want to compute a semi-global solution of (deterministic/stochastic) Hamilton Jacobi Bellmann (HJB) equation . For this purpose we use e.g. tree based tensor networks, in particular MPS/TT and/or Neural Networks. we focus first on an Lagrangian perspective which is related to dynamical programing and reinforcment learning. We consider control affine dynamical systems and quadratic cost for the control. For many high dimensional PDEs of practical interest, e.g. Backward Kolmogorov equations, HJB etc. the PDE operator cannot be easily expanded in tensor form. In this case, we propose machine learning approach confined to the manifold of tree based tensors with fixed multi-rank. In the present Lagrangian picture we apply policy iteration and solved the Linearized HJB by integrating along trajectories, defined by the corresponding dynamical system (characteristics) for samples of initial values. From the computed point values we infer the sought value function. In the stochastic case, we have many pathes instead of single trajectories. There the HJB can be reformulated by an (uncoupled) Forward Backward SDE system. The forward backward SDE is linked with (parabolic) PDEs by a non-linear Feynman-Kac theorem. For the present purpose we use e.g. tree based tensor networks, in particular MPS/TT and/or Neural Networks. L. Sallandt and coworkers have compared our tensor approach with already published results using deep neural networks (E & Jentzen et al., Pham et al.).

At the end we want to present an outlook of a new approach of an alternative Eulerian approach for Potential Mean Field Games.