contour line shows the same σ value. Locally we therefor have a situation analogous to Example 1, since we get the GE property for a whole piece of a contour line. Between BP_1 and the CCI point (0, 0) in addition to the maximal σ on the contour line indicating GE_3, we get two minimal σ points indicating the branches of GE_1 (Fig. 6c). On GE_3, the ray $y=-x$, we have the values

$$\sigma=(1+\tfrac{1}{2}x^2)^2$$

and on the contour line zero we have $\sigma=(1-x^2/2)^2$. Going out of SP_1, for an orthogonal trajectory we observe the same direction as for GE_1. Thus, GE_1 indicates the real valley over SP_1 here, and later GE_1 turns to the right with its valley ending by flattening in BP_1 on the slope of cirque v_2. The valley of GE_1 near BP_1 is so imperceptible that the gradient of the surface nearly suppresses its influence. Orthogonal trajectories flowing over the piece of the axis $y=x$ between the SP's (we see a dike) cross the GE_1 in cirque v_2 and the GE_2 on cliff r_2, almost without distortion. The reason is the possible divergence of the direction of gradient and of the vector tangential to a GE mentioned above.

Again in the CCI point we do not find any branching. In (0, 0) we have a nonzero gradient, but the Hessian

$$H=\begin{pmatrix}-y & y-x\\ y-x & x\end{pmatrix}$$

also has two zero eigenvalues, as in Example 2. So, here a "branching" condition for a steepest descent path is fulfilled [7]:

$$u_{gg}=0, \quad u_{tg}=0. \tag{8}$$

The CCI point lies on the straight contour line $y=x$ where we have the orthogonal gradient direction $(1,-1)$, except at the two SP's where the gradient vanishes. This is the direction of the GE_3 itself. In Example 2 we find the analogous gradient $(-1,1)$. Hence, there is no branching of an orthogonal trajectory, and it follows that condition equation (8) (Eq. (7) of [7]) is not sufficient. Orthogonal trajectories cannot bifurcate on a surface which is continuous and smooth, except at stationary points, cf [6]. In minima they meet asymptotically from nearly all directions along the direction of the smallest eigenvalue.

Example 4. We treat a modified HNR model

$$U(x,y)=\tfrac{1}{2}(xy^2-yx^2+\mu x^2+2y-3), \tag{9}$$

where we include a parameter μ. Here, $\mu=1$ is the HNR surface [1]. It is easy to execute derivations of the GE condition equation (3) with this model surface equation (9). It gives a polynomial of fifth degree in x and y. Even for $\mu=1$, it is not easy to factorize. Hence, the GE condition implicitly defines the GE curves. If we use the advantage of a 2D model, we can solve it by a point-by-point calculation over a close grid and draw contour curves, GE $(x,y)=$ constant, including the zero curve. This is done in Figs. 7–9 (cf. Fig. 2 in [1]). Model 4 is an asymmetric distortion of models 2 and 3. This results in a dramatic change

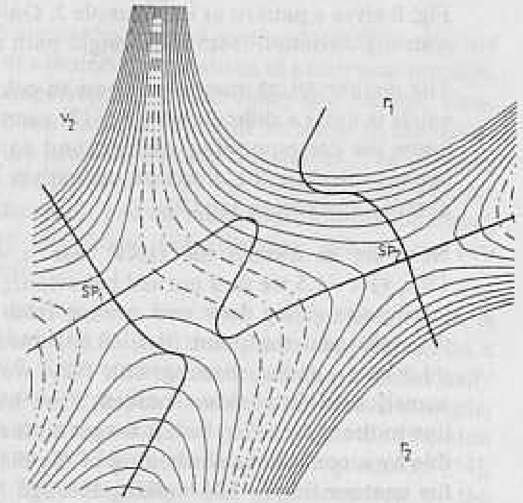

Fig. 7. Model surface $U(x,y)=\tfrac{1}{2}(xy(y-x)+1.15x^2+2y-3)$ including GE's, SP denotes a saddle point

of the GE behaviour in the central region of the surface. With the experience of Examples 2 and 3 we can give an explanation of the "strange" central pieces of the GE's in Figs. 7–9. In Fig. 8 we observe a straight GE_1 from south to north, i.e. from v_1 to r_1. The two other GE's cross GE_1 at true BP's of the type in Example 3, but with a nonorthogonal crossing angle. If we vary the parameter μ we shift mainly the SP_2. For $\mu>1.08$ it is raised, for $\mu<1.08$ it is lowered. In both cases we "lacerate" the BP's and only retain some turning points of the corresponding GE's. The turning point character of the central GE in the former

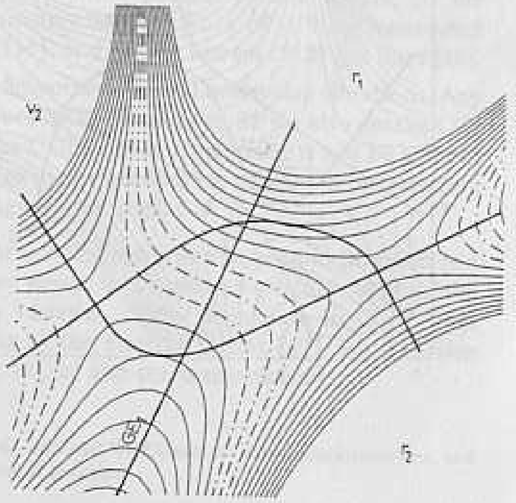

Fig. 8. Model surface $U(x,y)=\tfrac{1}{2}(xy(y-x)+1.08x^2+2y-3)$ including GE's